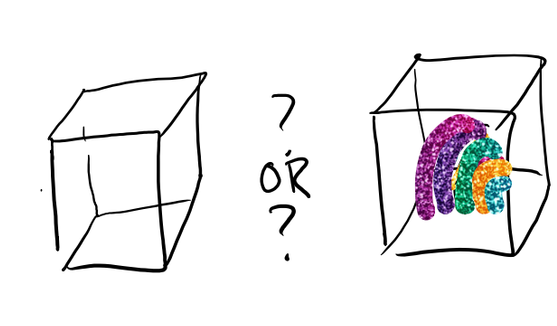
5: Bomb detection

Saturday, March 13, 2021   3:18 PM

Quantum Bomb Detection (Elitzur Vaidman Bomb Test)
(Rainbow)

You have a clear box. Nothing is in box, or a very sensitive rainbow bomb is in box, which will detonate a rainbow if a photon hits it.

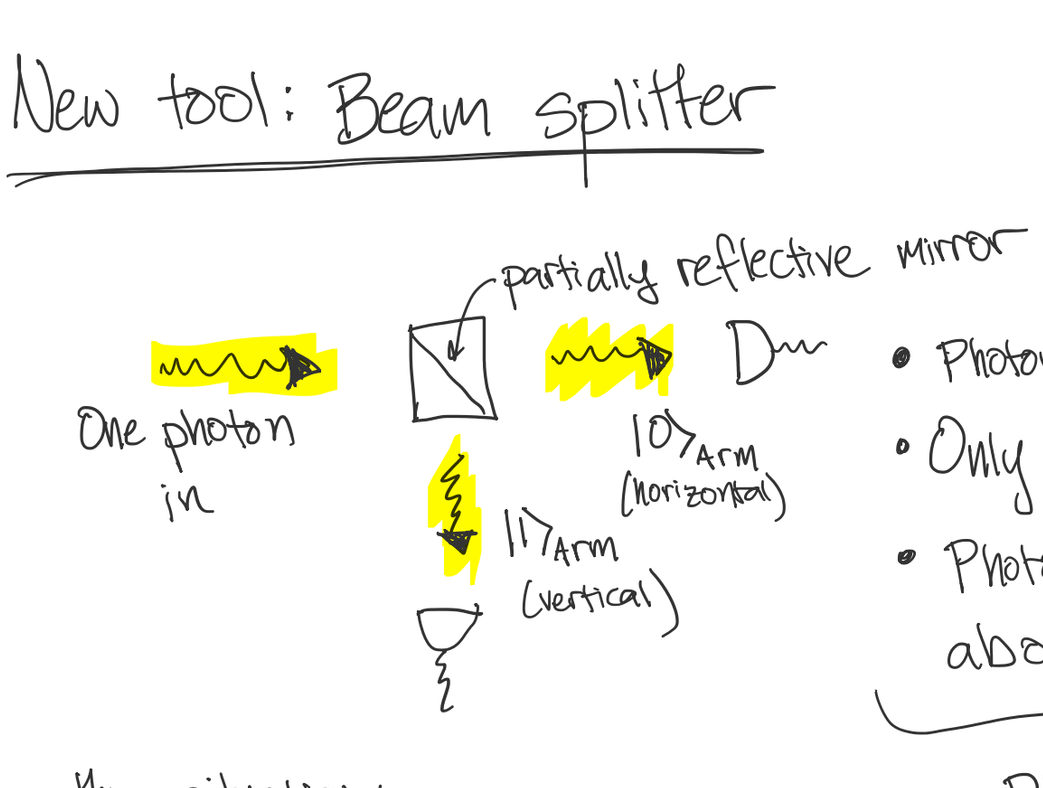


Shoot photon at box:
- No Bomb → photon comes out other side of box
- Bomb → photon triggers bomb → BOOM!

This is the only way you can interact with box.

You would like to give a rainbow bomb to your friend but you are worried you have a dud. What to do?

New tool: Beam splitter

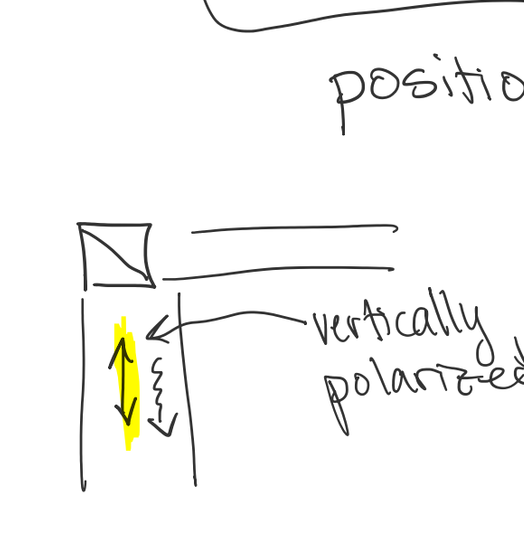


- Photon takes both paths in superposition
- Only one detector triggers
- Photon must "make decision" about its path (2 options)

position ⇒ qubit

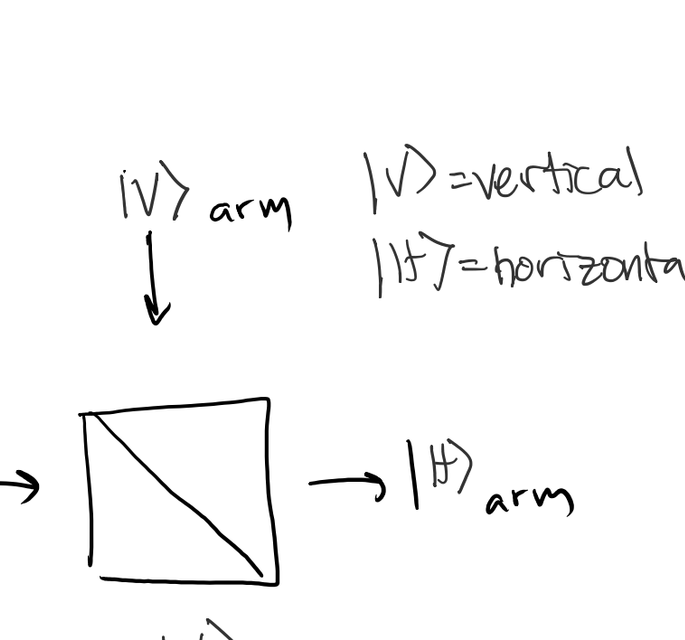
In this situation:
1 photon ⇝ 2 qubits

$|0\rangle_P |V\rangle_A$ means: polarization, arm



Beamsplitter Mirror → Transmission depends on polarization

- Transmits $|0\rangle_P$ (↕)
- Reflects $|1\rangle_P$ (⟷)



$|V\rangle$ = vertical
$|H\rangle$ = horizontal

Behavior

$|0\rangle_P|H\rangle_A$ → ⧄ → $|0\rangle_P|H\rangle_A$ (Pol) $|0\rangle_P|V\rangle_A$ → ⧄ → $|0\rangle_P|V\rangle_A$

$|1\rangle_P|H\rangle_A$ → ⧄ → $|1\rangle_P|V\rangle_A$ $|1\rangle_P|V\rangle_A$ → ⧄ → $|1\rangle_P|H\rangle_A$

ONE PHOTON!

$|H\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $|V\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

How does beamsplitter affect standard basis:

$|0\rangle_P|H\rangle_A \to |0\rangle_P|H\rangle_A$ ⟺ $\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \to \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$

$|0\rangle_P|V\rangle_A \to |0\rangle_P|V\rangle_A$ $\begin{pmatrix}0\\1\\0\\0\end{pmatrix} \to \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$

$|1\rangle_P|H\rangle_A \to |1\rangle_P|V\rangle_A$ $\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \to \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

$|1\rangle_P|V\rangle_A \to |1\rangle_P|H\rangle_A$ $\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \to \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$

What transforms vectors... matrices!

$$U_{BS} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

* only certain types of matrices are allowed. Will discuss which later

New part of Quantum Mechanics: Quantum Operations "Gates"

Note: Matrix is entirely defined by its action on standard states. For now, we will use this representation, and get into matrices later.

Other Gates

\ = 45° mirror $|H\rangle_A \to \backslash \to |V\rangle_A$ $|V\rangle_A \to \backslash \to |H\rangle_A$

(θ) = wave plate (rotates polarization) by θ $|0\rangle_P \to (45°) \to |+\rangle_P$ $|0\rangle_P \to (90°) \to |1\rangle_P$

Interferometer

Don't know how ⧄ acts on $|+\rangle$, only on $|0\rangle, |1\rangle$

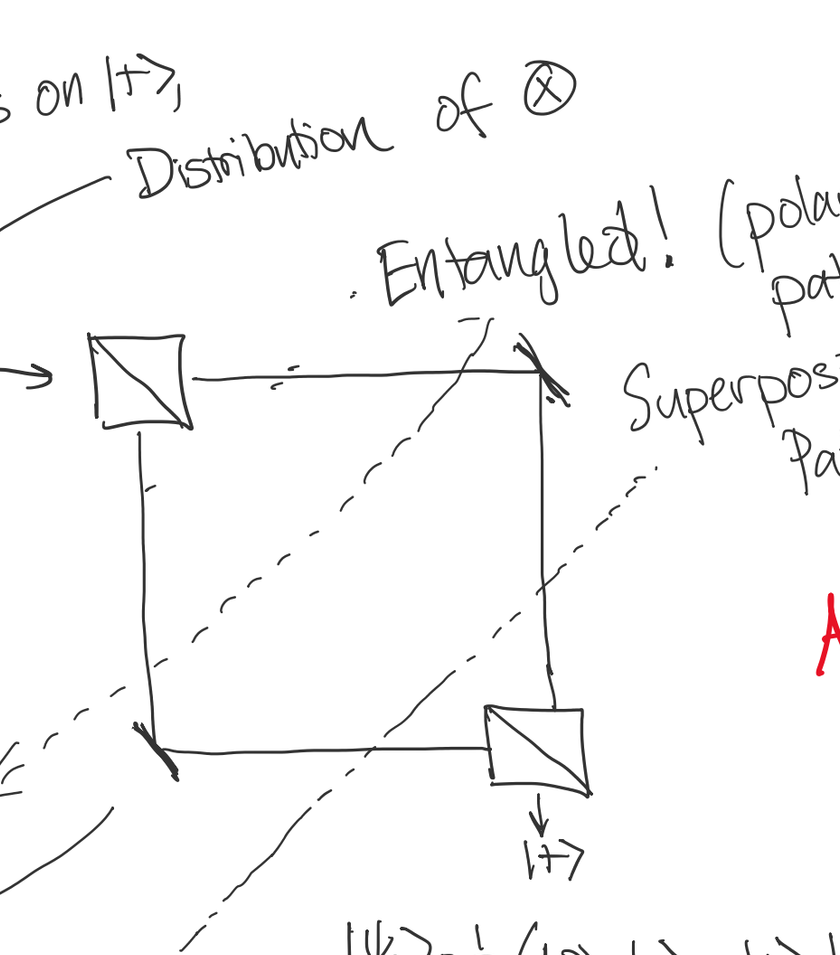
Distribution of ⊗

$$|\psi_0\rangle = |+\rangle_P|H\rangle_A = \frac{1}{\sqrt{2}}(|0\rangle_P|H\rangle_A + |1\rangle_P|H\rangle_A)$$

Action of ⧄ distributes b/c $M(\vec{v}_1 + \vec{v}_2) = M\vec{v}_1 + M\vec{v}_2$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle_P|H\rangle_A + |1\rangle_P|V\rangle_A)$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle_P|V\rangle_A + |1\rangle_P|H\rangle_A)$$



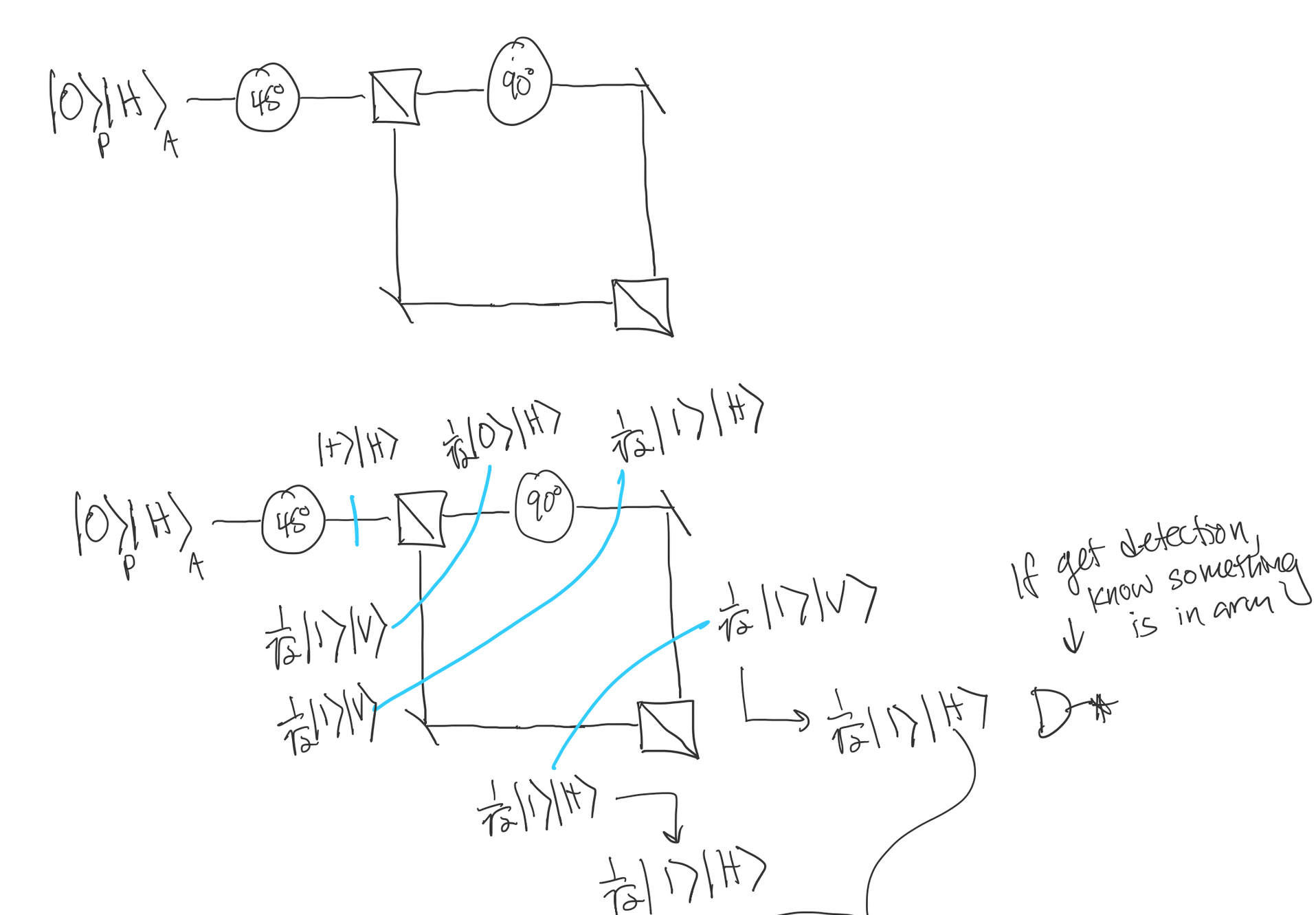
$$|\psi_f\rangle = \frac{1}{\sqrt{2}}(|0\rangle_P|V\rangle_A + |1\rangle_P|V\rangle_A) = \frac{1}{\sqrt{2}}(|0\rangle_P + |1\rangle_P)|V\rangle_A = \frac{1}{\sqrt{2}}|+\rangle|V\rangle_A$$

☆Photon is in superposition of paths. Path/Polarization are entangled. (Step through showing parts on path)

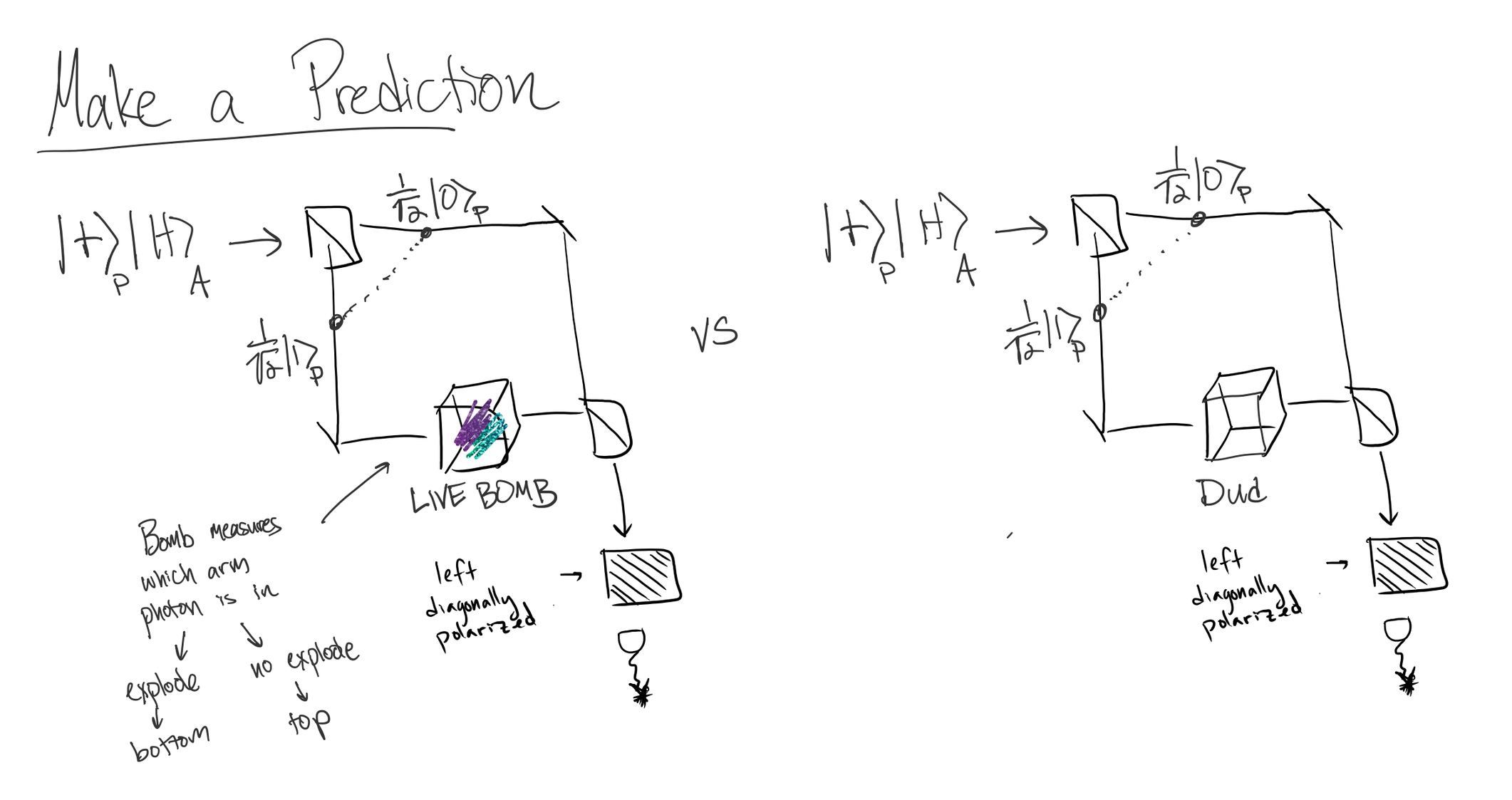
Output is very sensitive to differences b/t two paths.

Big Idea: Put bomb in one path. Less likely to explode, and maybe can learn something without exploding
If ask (measure) → only goes one path
If don't measure → goes both paths → $|+\rangle$

Group Work



Make a Prediction



Bomb is an Example of partial measurement, b/c bomb only measures A qubit, not P qubit. (A register, not P register)

In this class, we will only consider partial measurement in the standard basis. E.g. Bomb measures $|0\rangle$ vs $|1\rangle$, not $|+\rangle$ vs $|-\rangle$

Partial Measurement

$|\psi\rangle_{AB}$ (state w/ A register + B register)

To analyze result of partial measurement of register A in standard basis (Is A in state $|0\rangle$ or $|1\rangle$?)
(ex: have 2 photons, measure polarization of only 1, so far, we only talked about what happens if measure both.)

1. Ensure state is written in the following form: (Rewrite ex: $a|0\rangle|H\rangle + b|0\rangle|V\rangle$ ↴ $\frac{1}{\sqrt{2}}|0\rangle\left[(a|H\rangle + b|V\rangle)\sqrt{2}\right]$)

$$|\psi\rangle_{AB} = \alpha_0 |0\rangle_A |u\rangle_B + \alpha_1 |1\rangle_A |w\rangle_B$$

(See pset for how to put in form)

amplitudes; B register is in a normalized state; A register is in one of measurement basis states. Group terms

2. Result: Get outcome

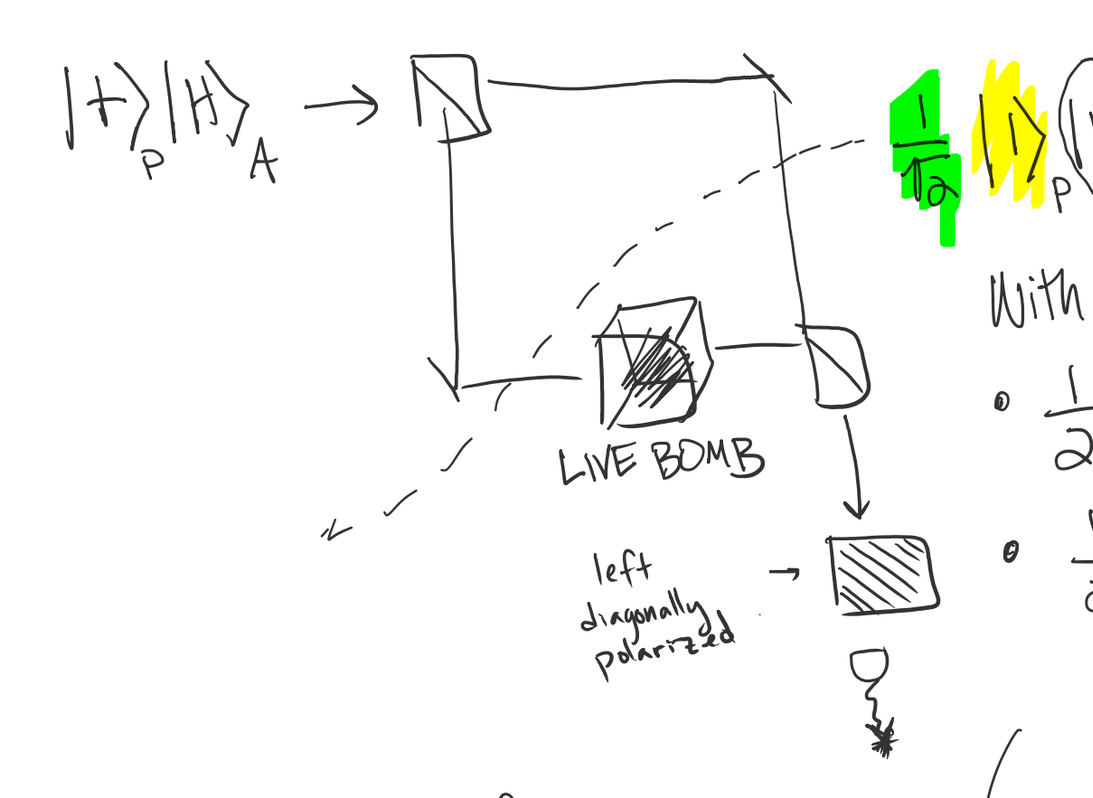
$|0\rangle$ with probability $|\alpha_0|^2$ / $|1\rangle$ with probability $|\alpha_1|^2$

State collapses to $|0\rangle_A|u\rangle_B$ / $|1\rangle_A|w\rangle_B$

☆No longer entangled!☆ Between A + B

Back to the Bomb

(already factored, already P is normalized)



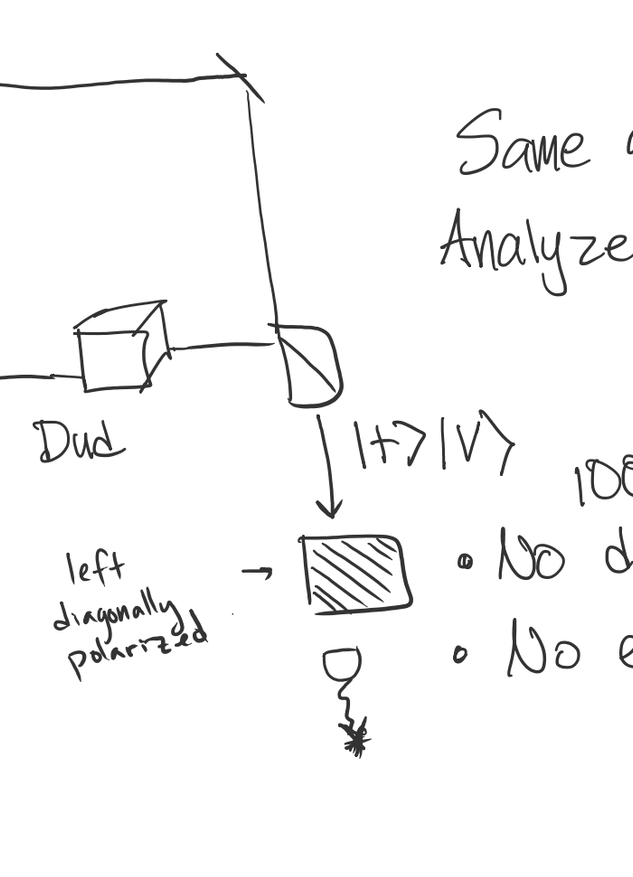
$$\frac{1}{\sqrt{2}}|1\rangle_P|H\rangle_A + \frac{1}{\sqrt{2}}|0\rangle_P|V\rangle_A$$

With probability
- $\frac{1}{2}$: $|1\rangle_P|H\rangle_A$ (explosion)
- $\frac{1}{2}$: $|0\rangle_P|V\rangle_A$ → No explosion

↓ $|0\rangle_P|V\rangle_A$ → ⧄ → ▨ → ½ prob of detection

Prob	Outcome
1/2	explosion
1/4	no explosion, detection
1/4	no explosion, no detection

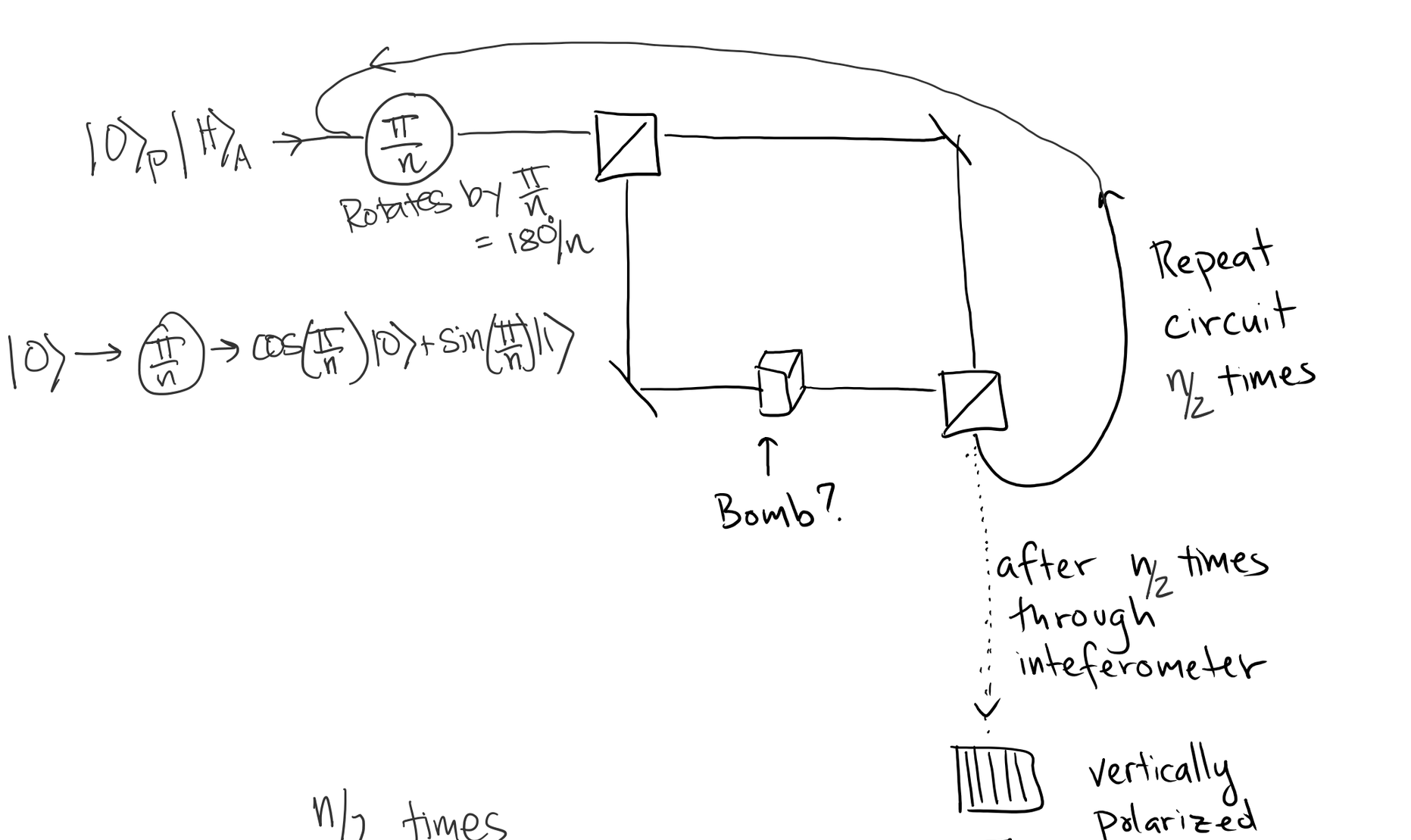
Dud: $|+\rangle_P|H\rangle_A$ →



Same as no bomb: Analyzed already!

100%:
- No detection
- No explosion

> If get detection, KNOW have live, unexploded bomb



Q: $|0\rangle \to (\frac{\pi}{n})$ (loop n/2 times; = 180°/n) → Polarization?

A) $|0\rangle$ B) $|1\rangle$ ↑ C) $|+\rangle$ D) $|-\rangle$

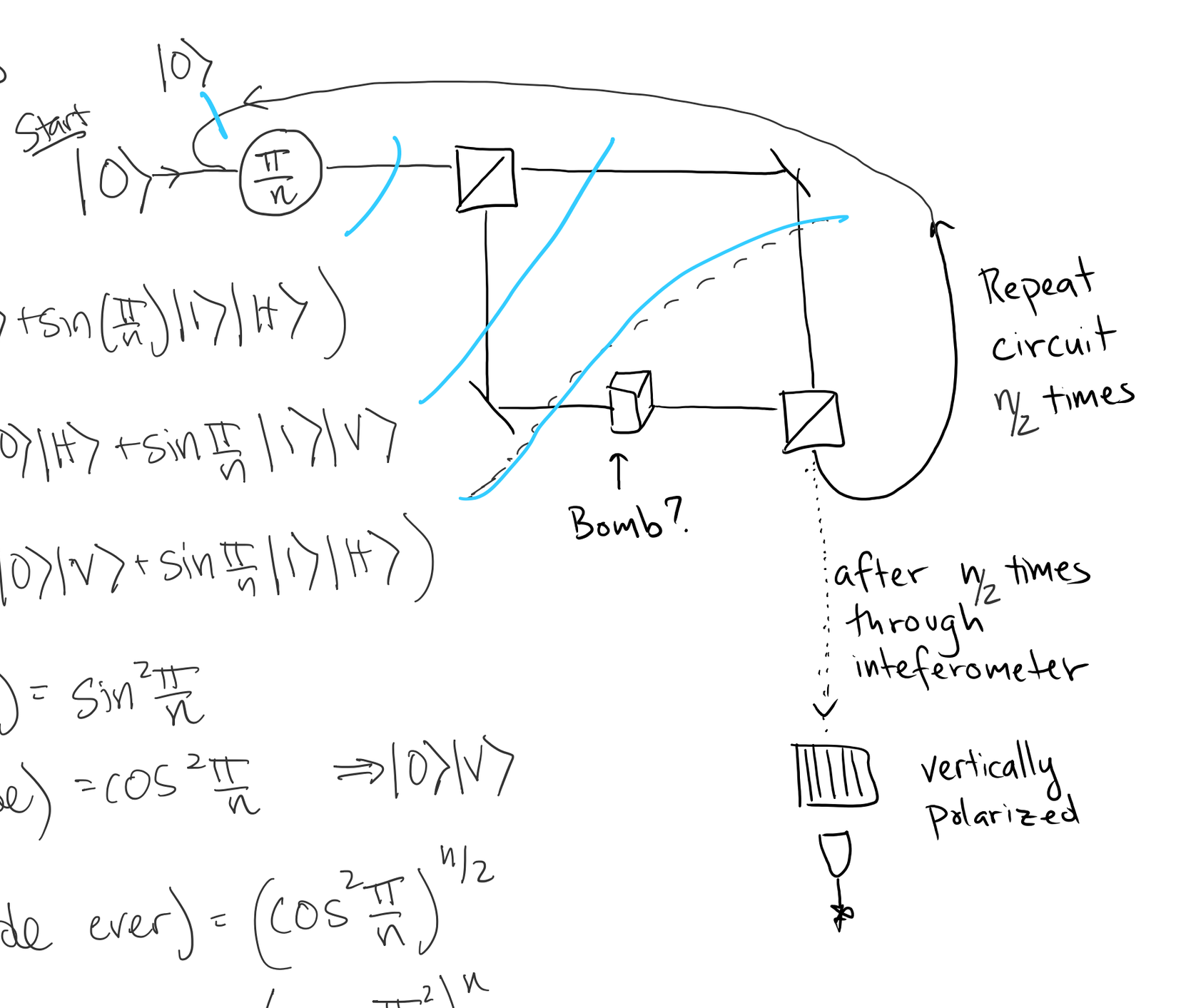
As a function of n:
1) If dud, what is probability of photon detection?
2) If bomb, what is probability of no explosion? (over entire run)
3) If bomb but no explosion, what is probability of detection?

$$*\left(\cos^2\frac{\pi}{n}\right)^{n/2} \approx 1 - \frac{\pi^2}{2n}$$

1) If dud:

- Effectively $|0\rangle_P \to (\frac{\pi}{n})$ (× n/2) → $|1\rangle$ ⟷
  No detection

2) If bomb



$$|0\rangle_P|H\rangle_A \to \left(\cos(\tfrac{\pi}{n})|0\rangle + \sin(\tfrac{\pi}{n})|1\rangle\right)|H\rangle$$
$$\to \left(\cos(\tfrac{\pi}{n})|0\rangle|H\rangle + \sin\tfrac{\pi}{n}|1\rangle|V\rangle\right)$$
$$\to \left(\cos\tfrac{\pi}{n}|0\rangle|V\rangle + \sin\tfrac{\pi}{n}|1\rangle|H\rangle\right)$$

$Pr(\text{explodes}) = \sin^2\frac{\pi}{n}$

$Pr(\text{no-explode}) = \cos^2\frac{\pi}{n}$ ⇒ $|0\rangle|V\rangle$

$$Pr(\text{no-explode ever}) = \left(\cos^2\frac{\pi}{n}\right)^{n/2} \approx \left(1 - \frac{\pi^2}{2n^2}\right)^n \approx 1 - \frac{\pi^2}{2n}$$

If no explode always $|0\rangle|V\rangle$ when exits. ⇒ Detection! 100% (If no explode)

A note on measurement:

$\{|0\rangle, |1\rangle\}$ ⇒ "Measurement" "Measurement Basis"
$|0\rangle$ ⇒ possible measurement outcome

$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ ⇒ "Measurement"
$|10\rangle$ ⇒ possible measurement outcome